16th December 2015

Majorana Fermions in Superconducting Chains

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Introduction

Fermions (I)

Quantum many-body theory:

- Fermions $$|\psi(x_1, x_2)\rangle = -|\psi(x_2, x_1)\rangle$$
- Bosons $$|\psi(x_1, x_2)\rangle = |\psi(x_2, x_1)\rangle$$

Fermions (II)

Properties

Pauli exclusion principle

$$|\psi(x_1, x_1)\rangle = -|\psi(x_1, x_1)\rangle$$

Properties

Pauli exclusion principle

$$|\psi(x_1, x_1)\rangle = -|\psi(x_1, x_1)\rangle$$

$$(c_i^\dagger)^2 = c_i^2 = 0$$

Properties

Pauli exclusion principle

$$|\psi(x_1, x_1)\rangle = -|\psi(x_1, x_1)\rangle$$

Anticommutation relations

$$|\psi(x_1, x_2)\rangle = c_1^\dagger c_2^\dagger |0\rangle$$

$$|\psi(x_2, x_1)\rangle = c_2^\dagger c_1^\dagger |0\rangle$$

Properties

Pauli exclusion principle

$$|\psi(x_1, x_1)\rangle = -|\psi(x_1, x_1)\rangle$$

Anticommutation relations

$$c_1^\dagger c_2^\dagger |0\rangle = -c_2^\dagger c_1^\dagger |0\rangle$$

Properties

Pauli exclusion principle

$$|\psi(x_1, x_1)\rangle = -|\psi(x_1, x_1)\rangle$$

Anticommutation relations

$$c_1^\dagger c_2^\dagger|0\rangle = -c_2^\dagger c_1^\dagger|0\rangle$$

$$c_1^\dagger c_2^\dagger + c_2^\dagger c_1^\dagger \equiv \left\{c_1^\dagger, c_2^\dagger\right\} = 0$$

Fermions (II)

Properties

Pauli exclusion principle

$$|\psi(x_1, x_1)\rangle = -|\psi(x_1, x_1)\rangle$$

Anticommutation relations

$$\{c_i, c_j\} = \left\{c_i^\dagger, c_j^\dagger\right\} = 0 \qquad \left\{c_i, c_j^\dagger\right\} = \delta_{i,j}$$

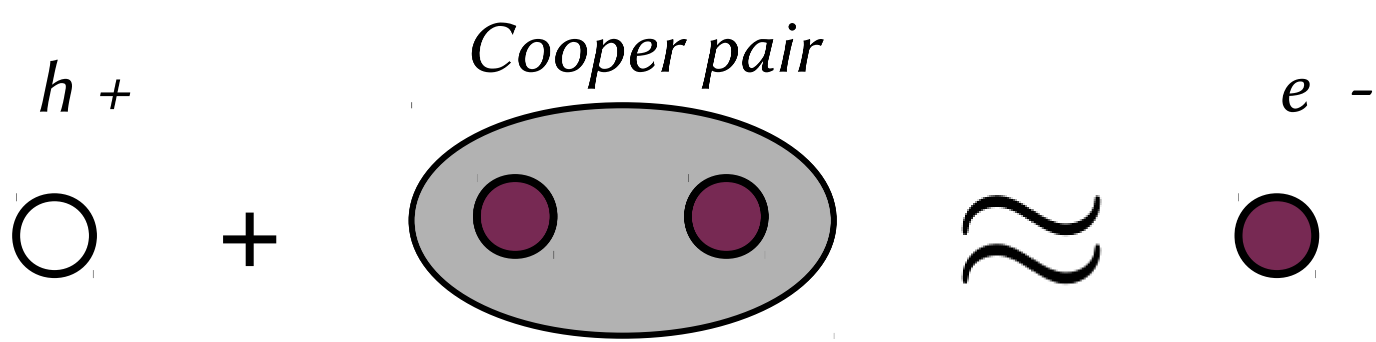
Properties

Hermitian creation operators $\gamma = \gamma^\dagger$

Chargeless → linear combination of electron and hole

Where to look for Majorana fermions?

Superconductivity



Kitaev wire

Finite superconducting chain

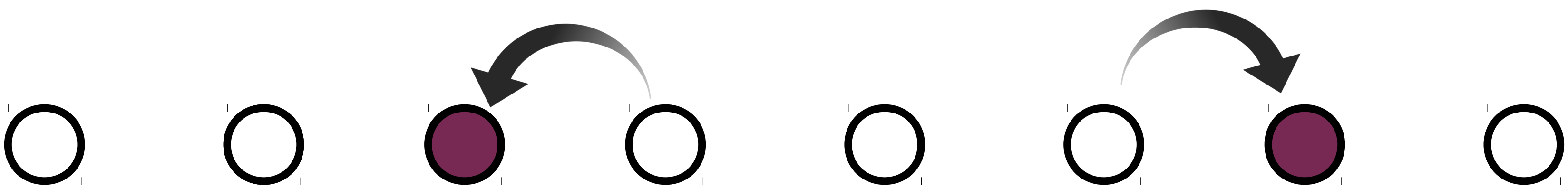
A.Kitaev, *Unpaired Majorana fermions in quantum wires,* Physics-Uspekhi, 2001

Kitaev wire

site occupation

$$\mathcal{H} = \boxed{-\mu \sum_{n}^{N} c_n c_n^\dagger}$$

Kitaev wire



hopping between neighboring sites

$$\mathcal{H} = -\sum_{n}^{N} \mu \, c \, c^{\dagger} - \sum_{n}^{N-1} (t \, c_n^{\dagger} \, c_{n+1} + \; h.c.)$$

Kitaev wire

addition of a cooper pair in neighboring sites

$$\mathcal{H} = -\sum_{n}^{N} \mu\, c\, c^{\dagger} - \sum_{n}^{N-1} (t\, c_n^{\dagger}\, c_{n+1} - \boxed{\Delta\, c_n\, c_{n+1}} + h.c.)$$

Majorana operators

$$\gamma_1 = \frac{c + c^\dagger}{2} \qquad \gamma_2 = i\frac{c - c^\dagger}{2}$$

- Hermitian operators
- Majorana from different sites satisfy fermion commutation relations
- Two Majorana operators correspond to one fermion

Majorana operators

$$\gamma_{2n-1} = \frac{c_n + c_n^\dagger}{2} \qquad \gamma_{2n} = i\frac{c_n - c_n^\dagger}{2}$$

- Hermitian operators

- Majorana from different sites satisfy fermion commutation relations

- Two Majorana operators correspond to one fermion

Finite superconducting chain

Kitaev hamiltonian

Analytically solvable cases:

- $t = \Delta = 0, \mu \neq 0$

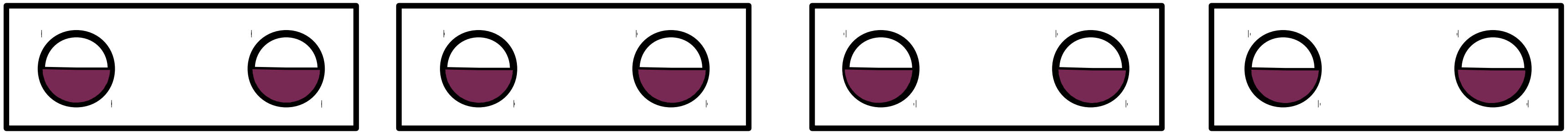
Trivial phase $$\mathcal{H} = i\mu \sum_{n=1}^{N} \gamma_{2n-1}\gamma_{2n}$$

- $\mu = 0, t = -\Delta \neq 0$

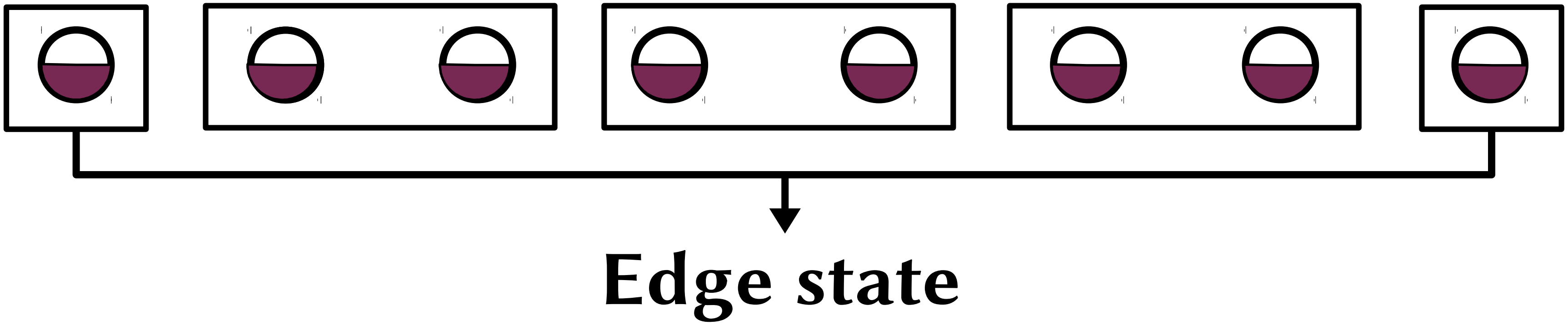
Topological phase $$\mathcal{H} = it \sum_{n=1}^{N-1} \gamma_{2n}\gamma_{2n+1}$$

Kitaev hamiltonian

- $t = \Delta = 0, \mu \neq 0$

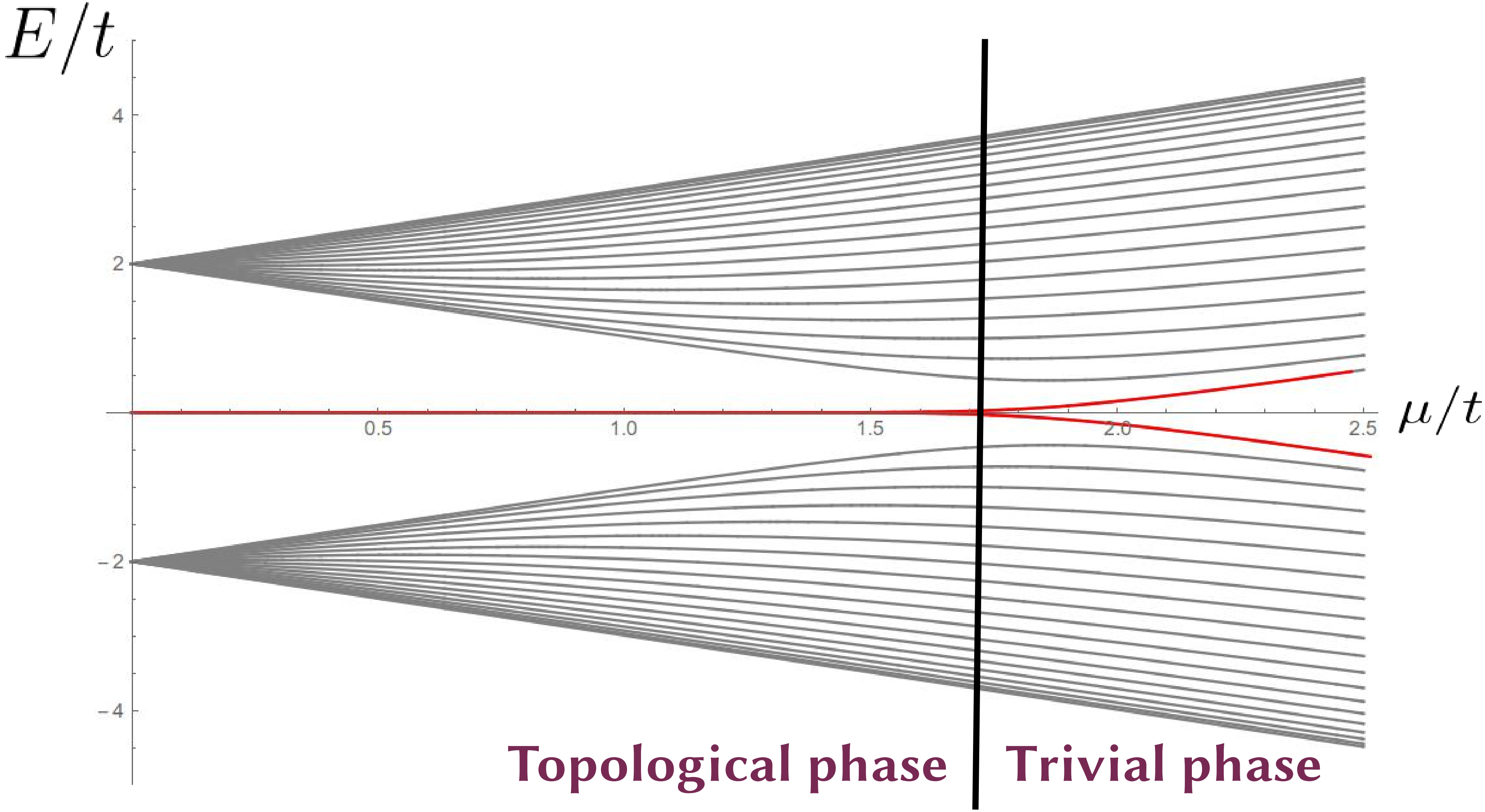


- $\mu = 0, t = -\Delta \neq 0$



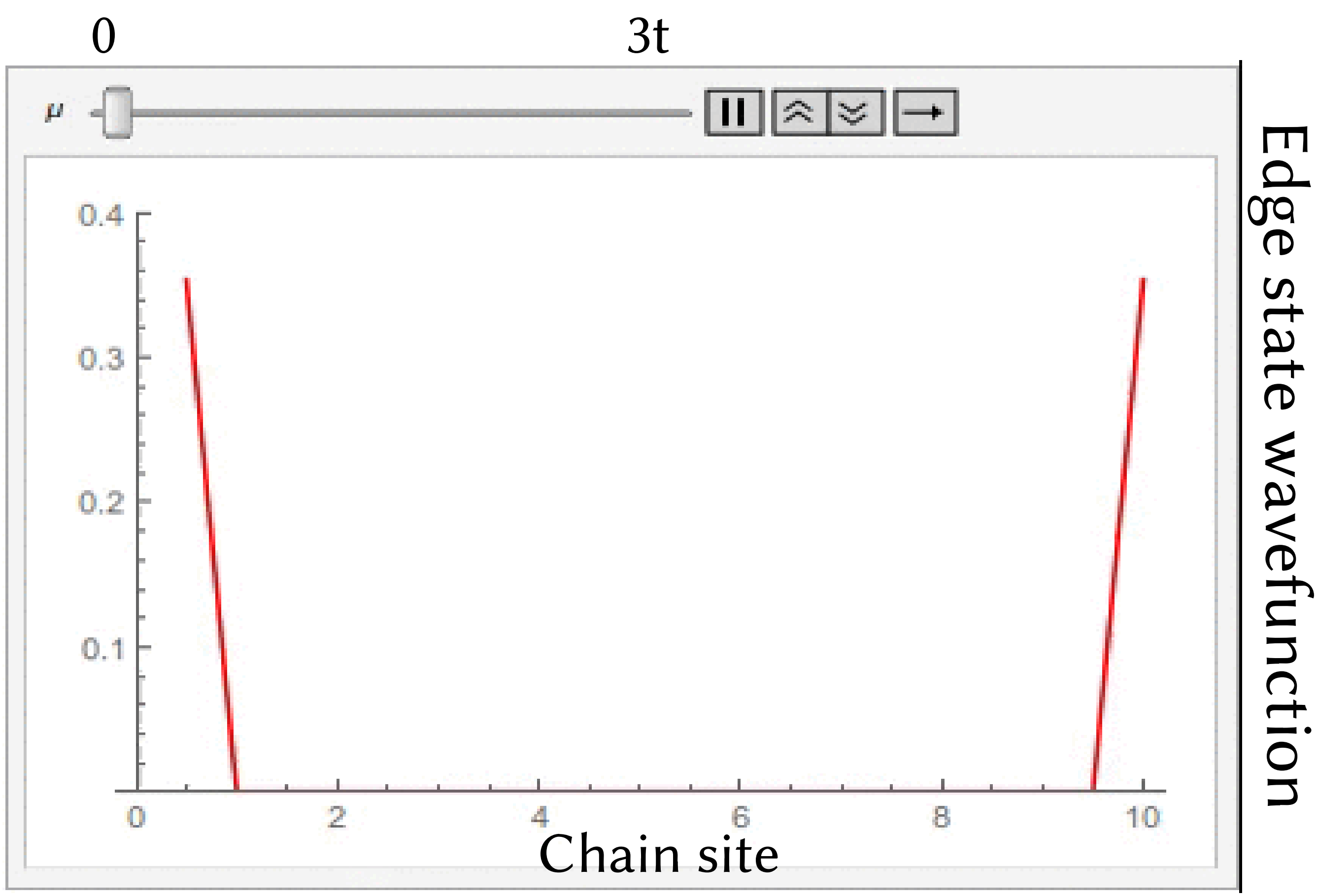
Numerical calculation

$$t = -1, \Delta = -1, N = 20$$



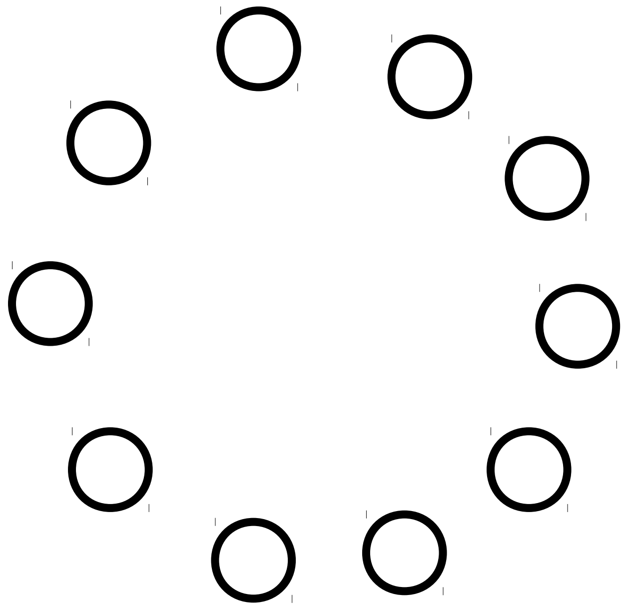
Numerical calculation

$$t = -1, \Delta = -1, N = 20$$



Infinite superconducting chain

Closing the chain



Periodic boundary conditions

Fourier transform into momentum space

Momentum space

$$|k\rangle = N^{-1/2} \sum_{n}^{N} e^{-ikn} |n\rangle$$

Transformation

$$\mathcal{H}(k) = \sum_{k} \langle k|H|k\rangle$$

Hamiltonian becomes decomposable

$$\langle k|H|k\rangle = (-2tcos(k) - \mu)\tau_z + \Delta sin(k)\tau_y$$

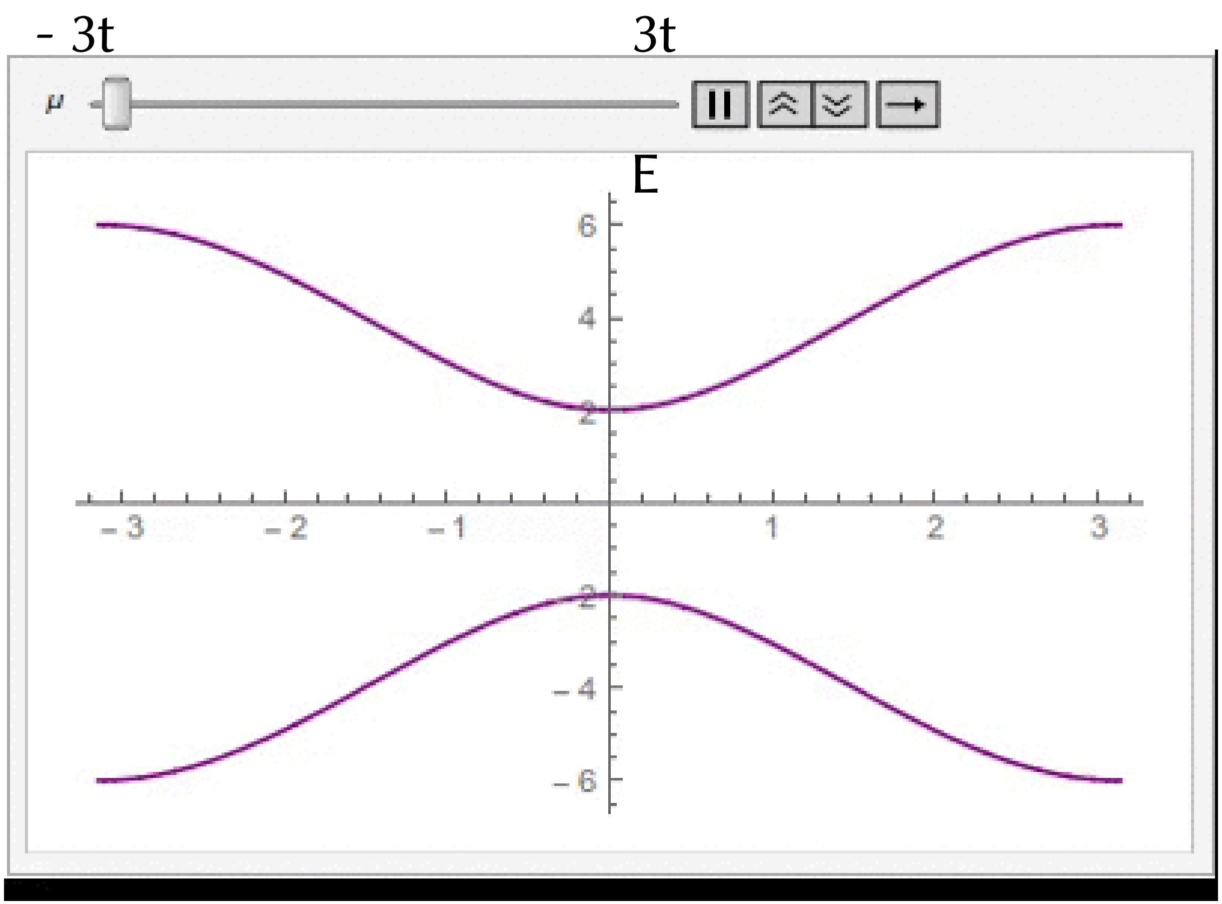
Momentum space

$$\langle k|H|k\rangle = (-2tcos(k) - \mu)\tau_z + \Delta sin(k)\tau_y$$

$$\tau_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad \tau_y = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$$

Problem becomes 2-dimentional

Energy bands



Gap closing at μ = -2t and μ = 2t

Topological invariant

Quantity that can identify the topological phase of the system

$$\mathcal{H} = \bar{\sigma} \cdot \bar{h}(k) \qquad \bar{\sigma} = \left(\tau_x, \quad \tau_y, \quad \tau_z\right)$$

Topological invariant

Quantity that can identify the topological phase of the system

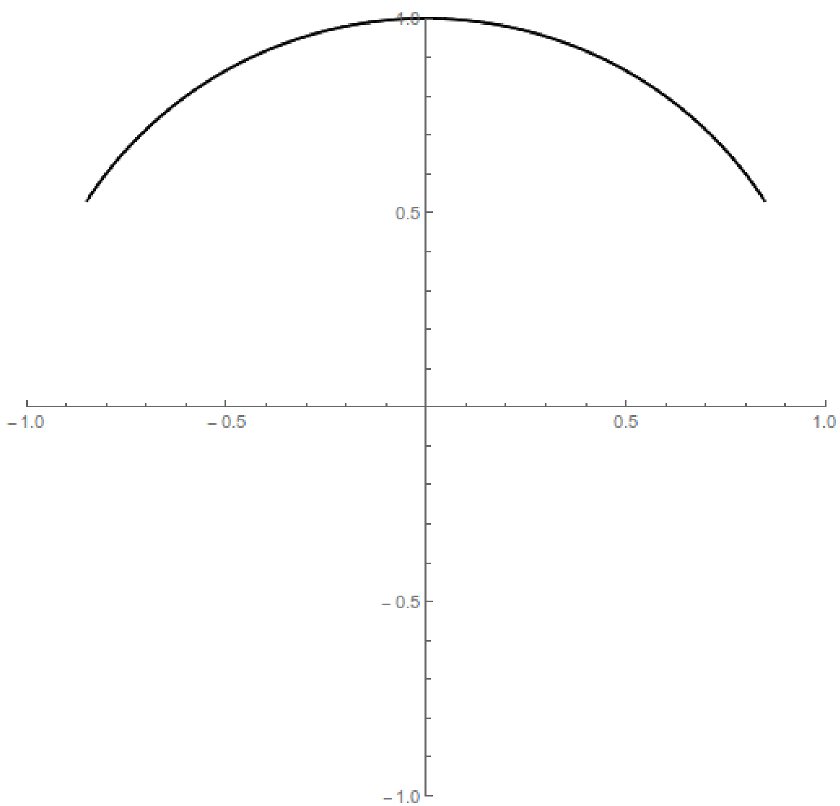
$$\mathcal{H} = \bar{\sigma} \cdot \bar{h}(k) \qquad \bar{\sigma} = \left(\tau_x, \quad \tau_y, \quad \tau_z\right)$$

$$\langle k|H|k\rangle = (-2tcos(k) - \mu)\tau_z + \Delta sin(k)\tau_y$$

$$\bar{h}(k) = \left(0, \quad \Delta sin(k), \quad -2tcos(k) - \mu\right)$$

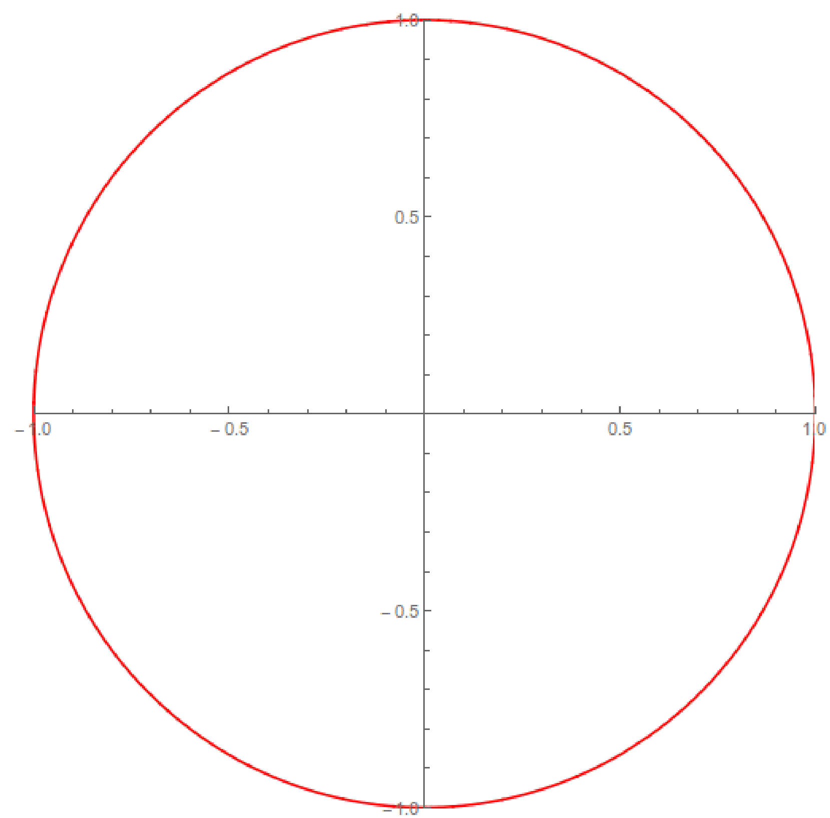
Trivial phase

$k = 0$ $k = \pi$



Topological phase

$k = 0$



$k = \pi$

Topological invariant

Invariant quantity

$$Q = sgn(h_z(0)h_z(\pi))$$

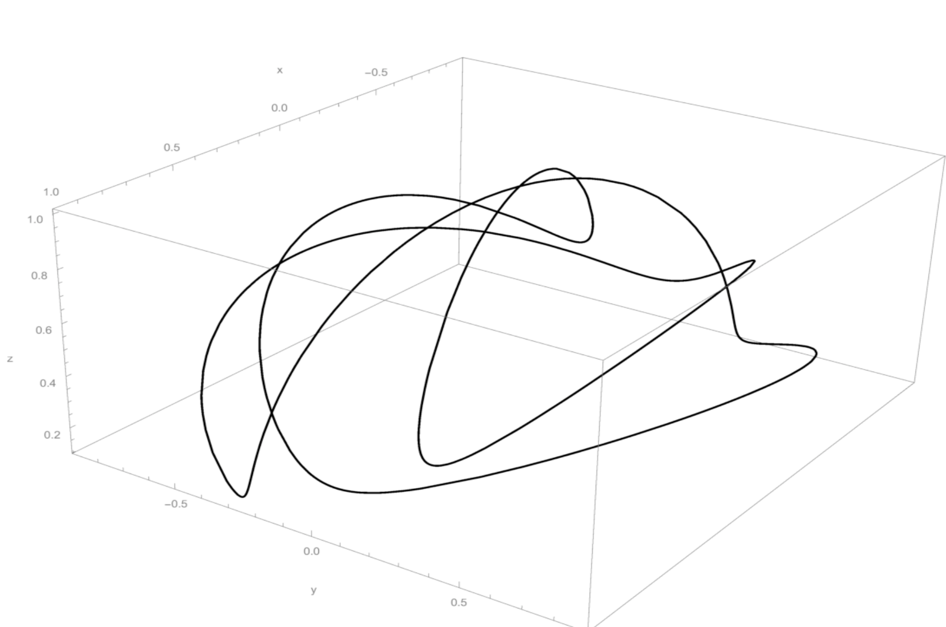
$$Q = \begin{cases} 1 & \text{In the trivial phase} \\ -1 & \text{In the topological phase} \end{cases}$$

More general Hamiltonian

$$h_{x,y}(k) = -h(-k) \qquad h_z(k) = h_z(-k)$$

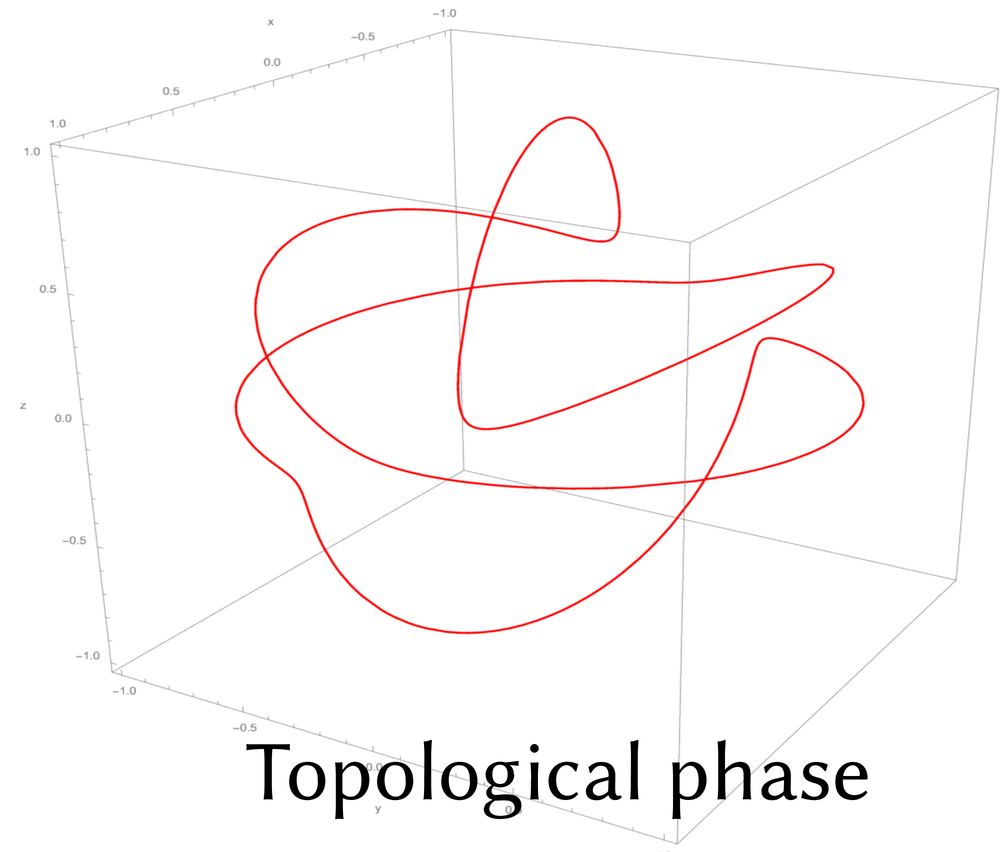
Topological invariant

$$h_x(k) = sin(3k) \quad h_y(k) = 2\Delta sin(2k) \quad h_z(k) = 2tcos(2k) - \mu$$



Trivial phase

$Q = 1$



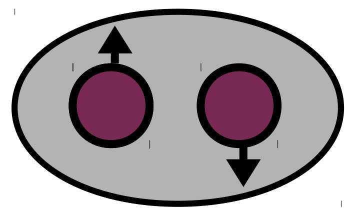
Topological phase

$Q = -1$

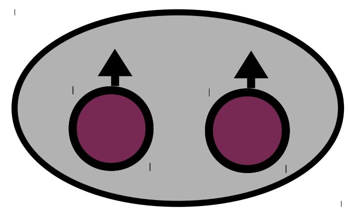
Experimental realization and results

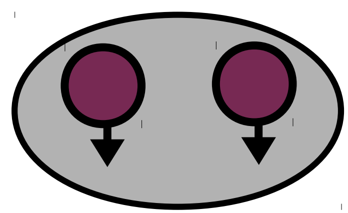
s & p superconductors

- S-pairing



- P-pairing





System requirements

Spinless p-wave **superconductor**

↓

Band gap and cooper pairing

System requirements

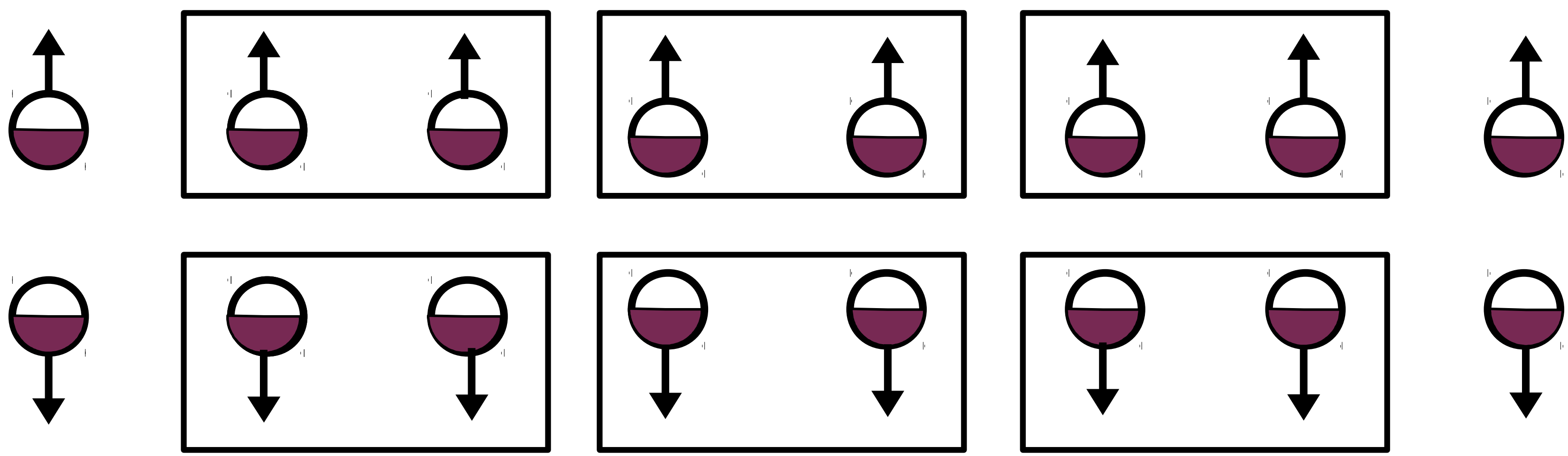
Spinless **p-wave** superconductor

↓

Allows for momentum dependent band gap

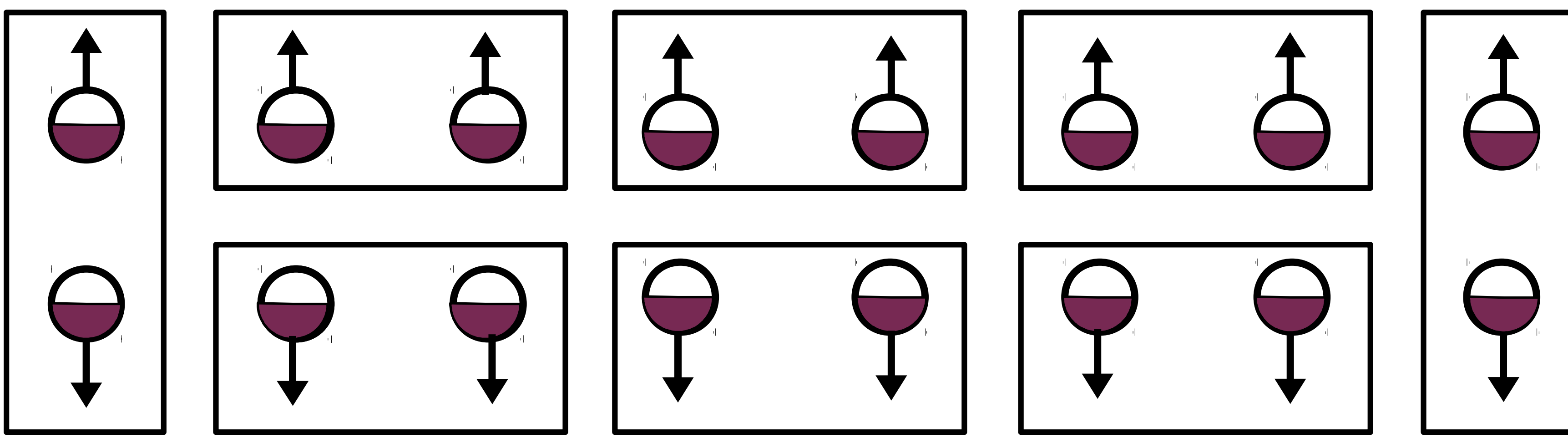
System requirements

Spinless p-wave superconductor



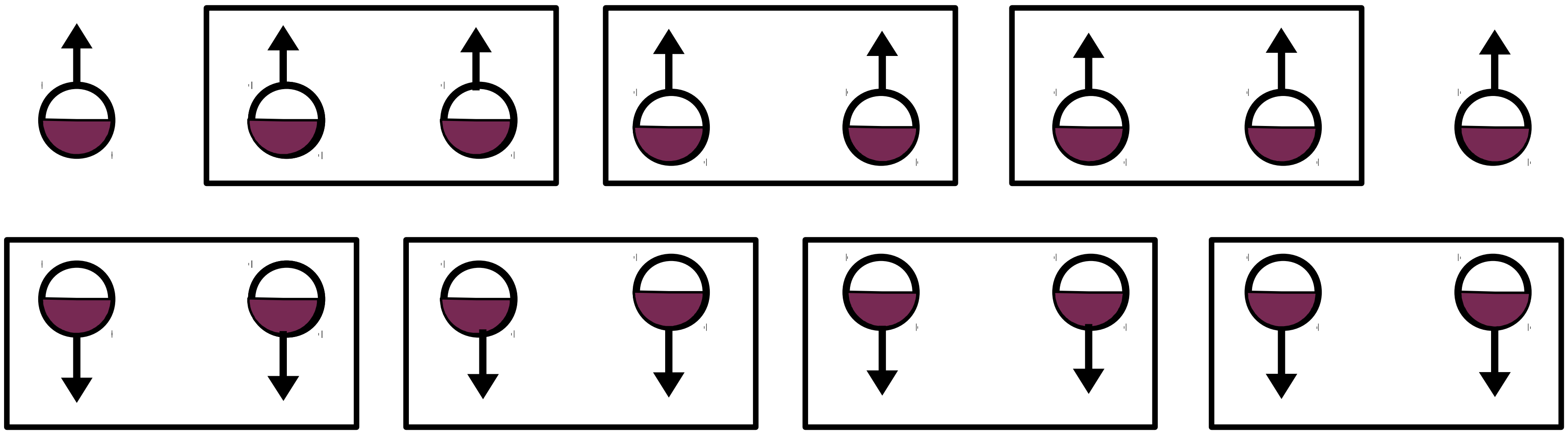
System requirements

Spinless p-wave superconductor



System requirements

Spinless p-wave superconductor

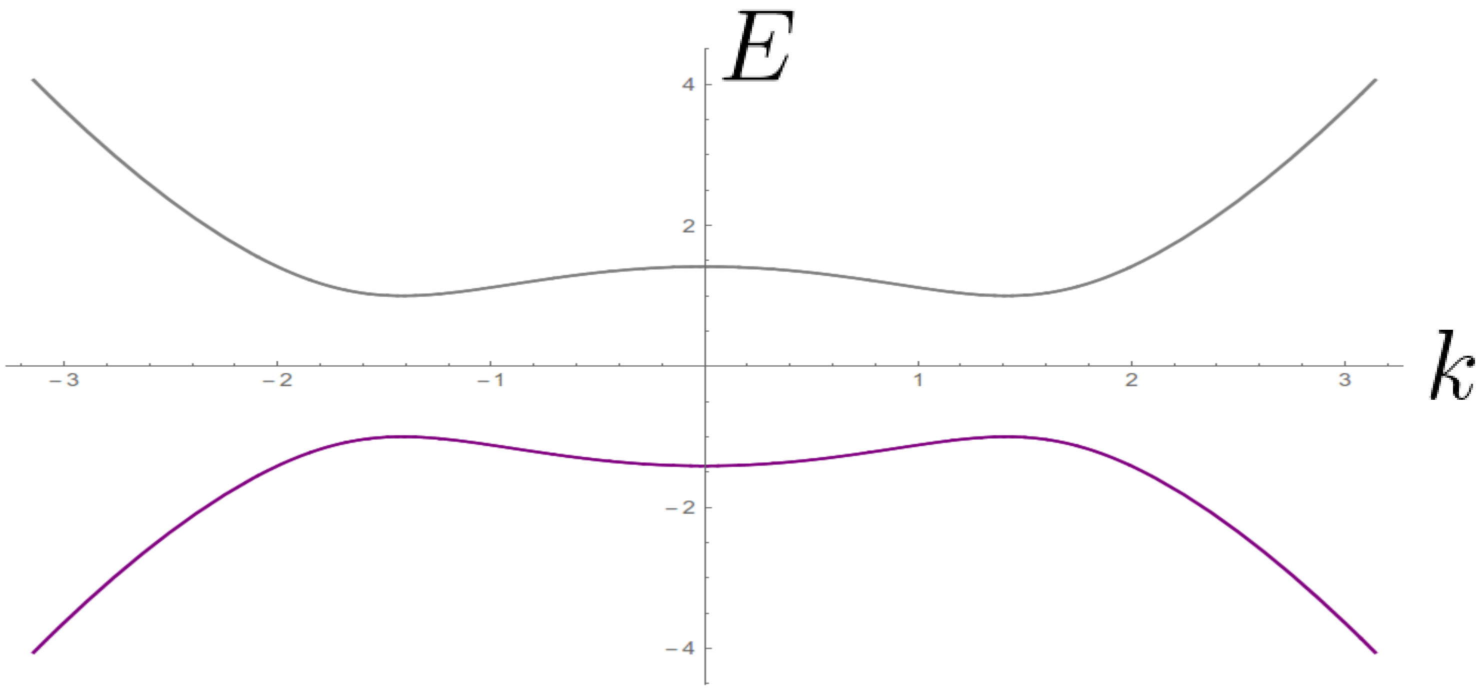


System requirements

- Superconductivity
- Magnetic field
- Spin orbit coupling

System requirements

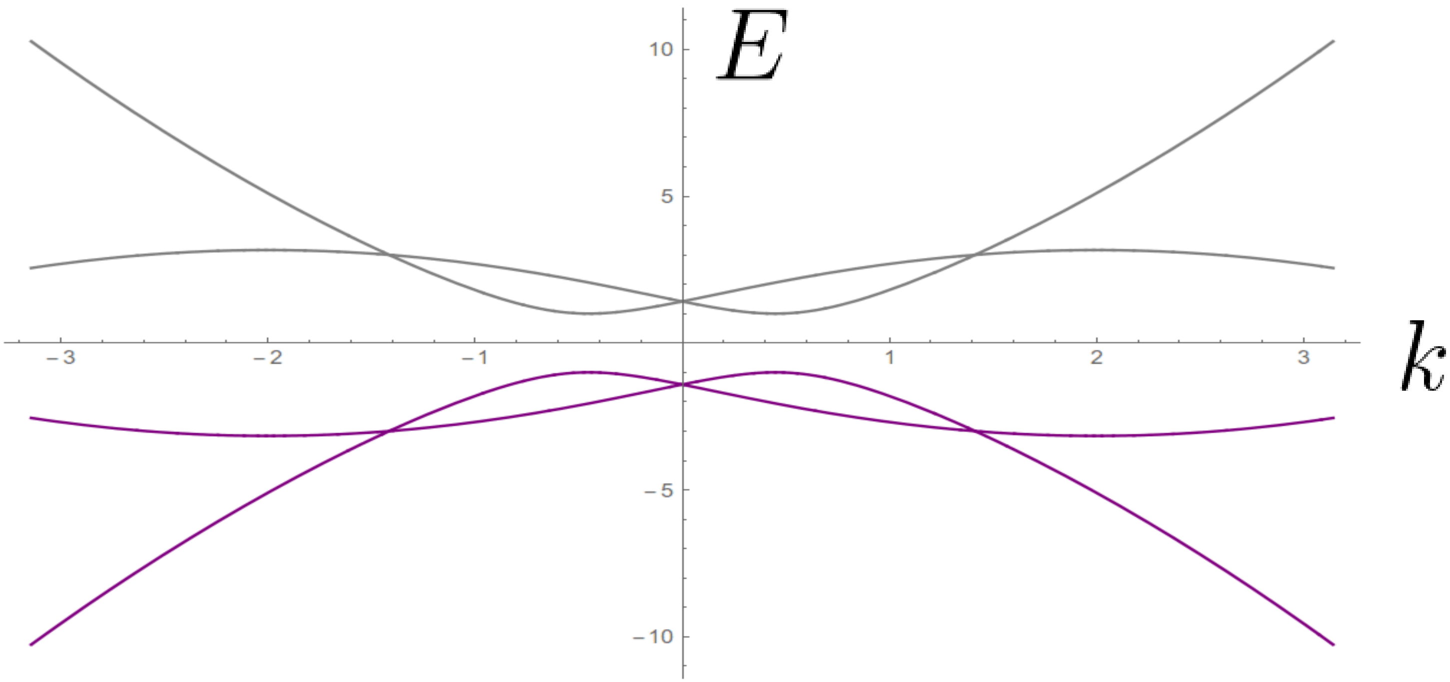
Electron chain + superconductivity



$$\mathcal{H} = \boxed{\frac{k^2}{2m}\tau_z + \Delta\tau_x}$$

System requirements

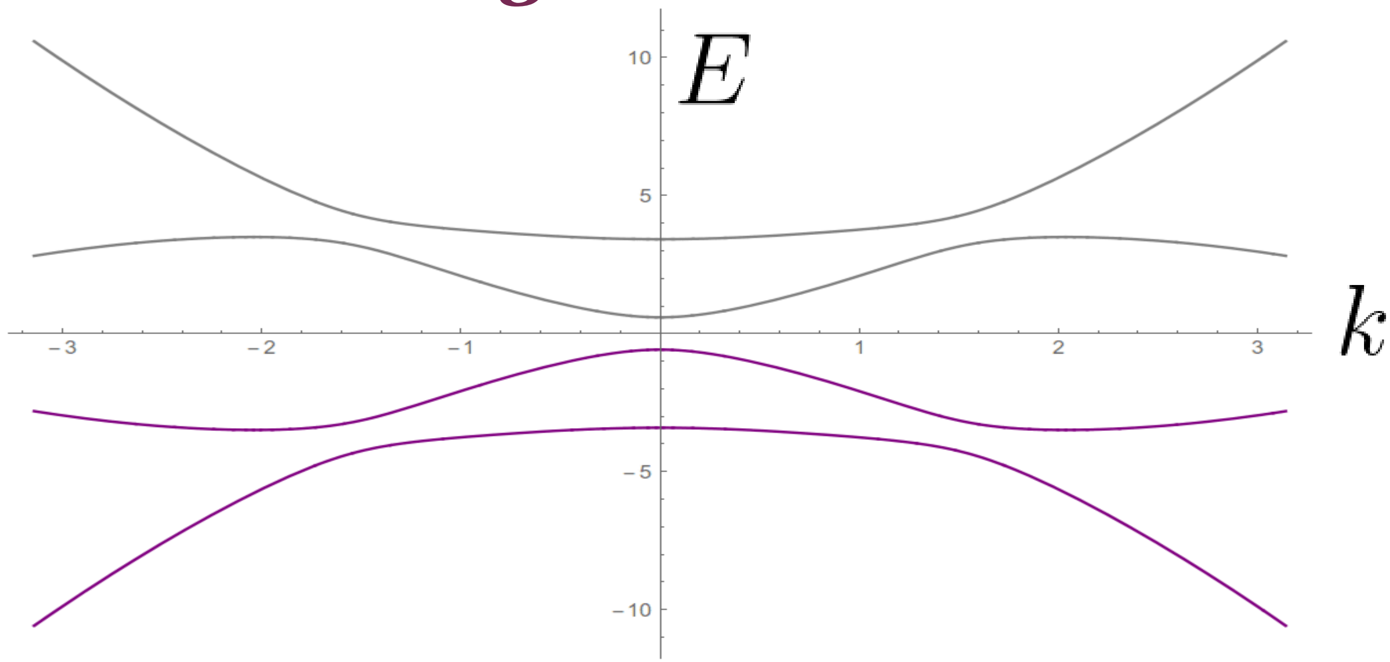
Electron chain + superconductivity + **spin orbit coupling**



$$\mathcal{H} = \left(\frac{k^2}{2m} + \boxed{\alpha \sigma_y k} \right) \tau_z + \Delta \tau_x$$

System requirements

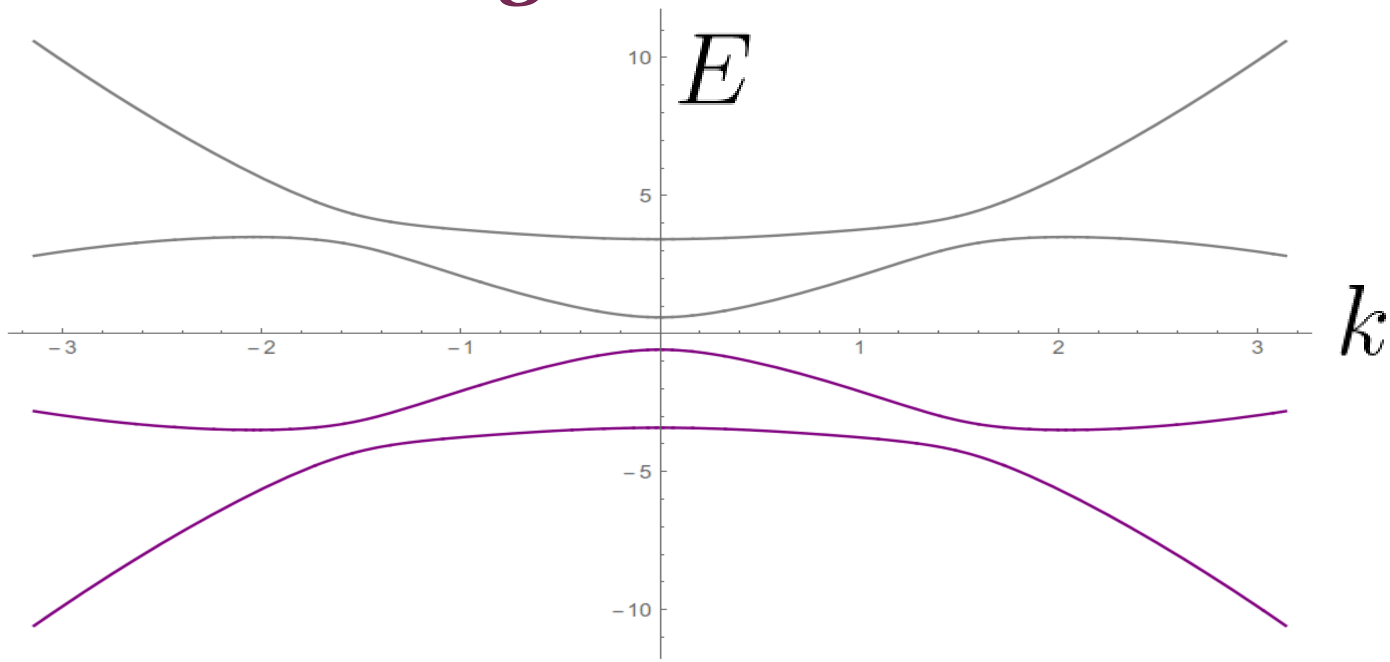
Electron chain + superconductivity + spin orbit coupling + **magnetic field**



$$\mathcal{H} = \left(\frac{k^2}{2m} + \alpha\sigma_y k \right)\tau_z + \boxed{B\sigma_z} + \Delta\tau_x$$

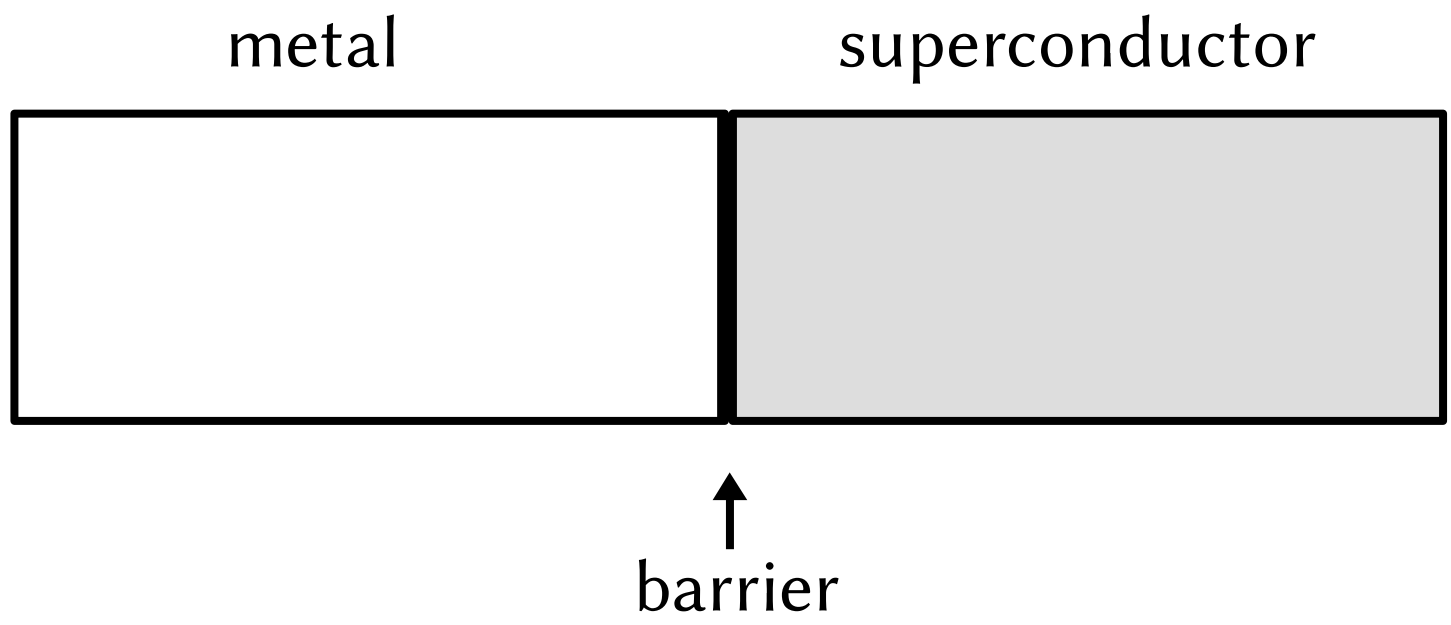
System requirements

Electron chain + superconductivity + spin orbit coupling + **magnetic field**

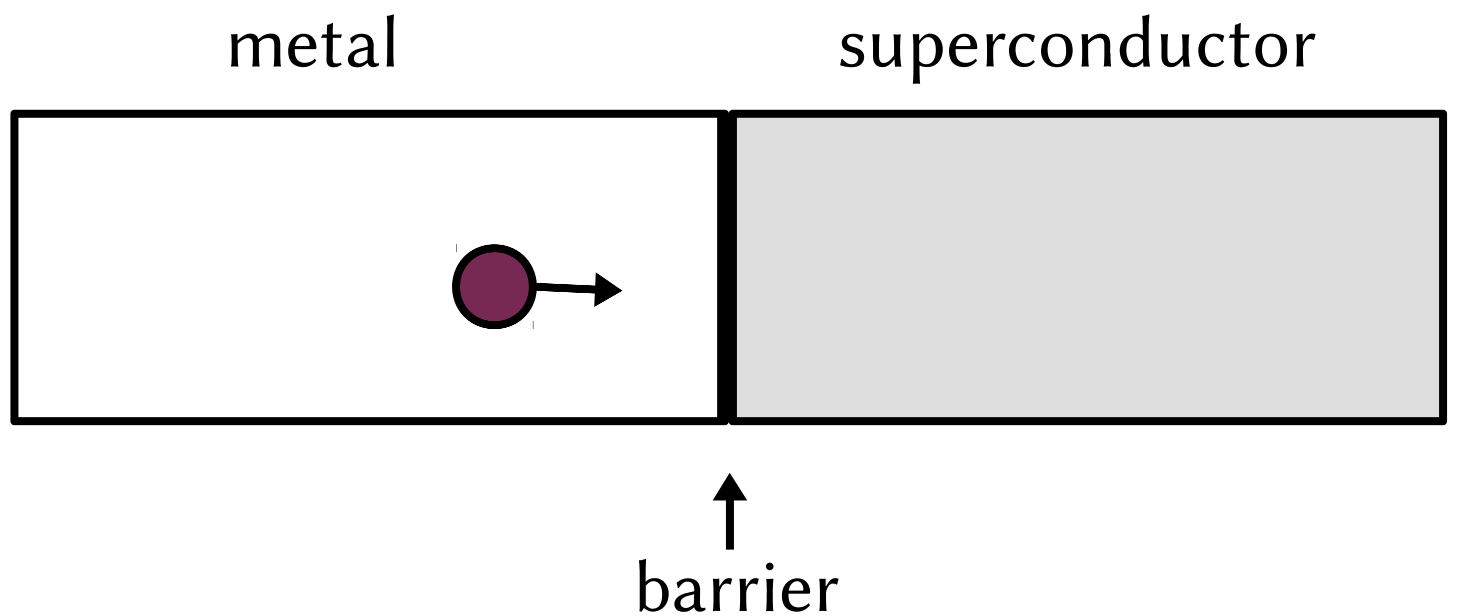


Condition: $B^2 > \Delta^2 + \mu^2$

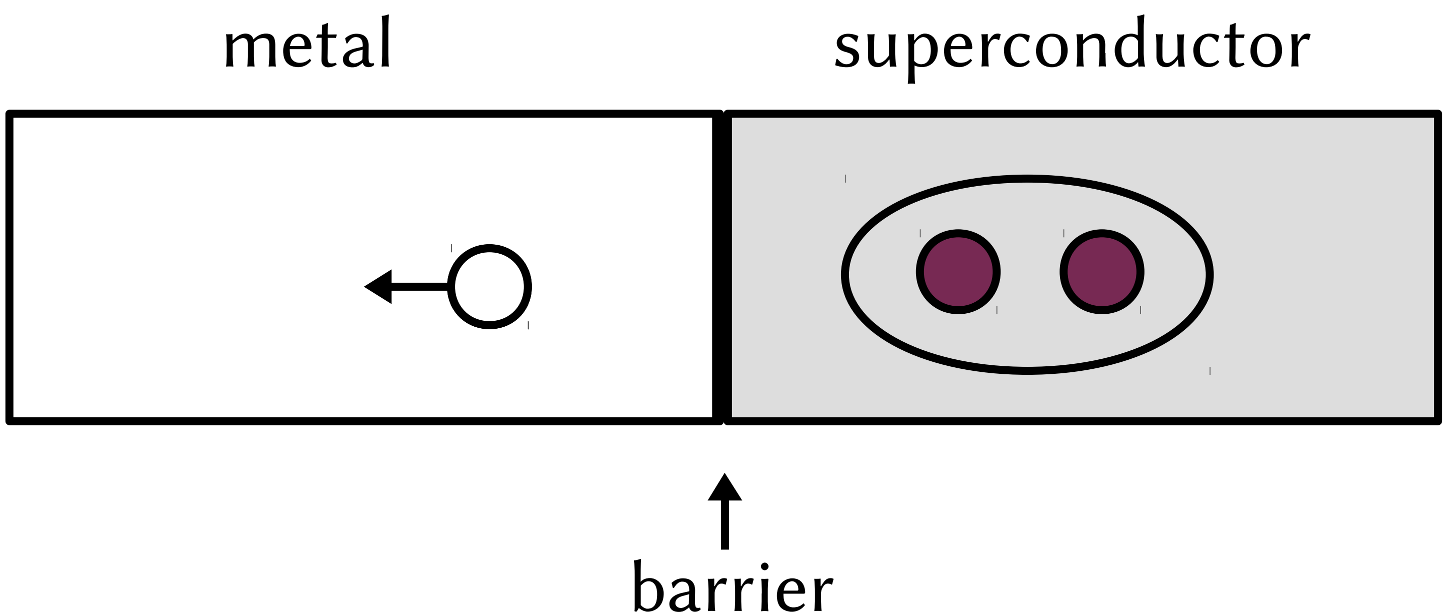
Andreev reflection



Andreev reflection

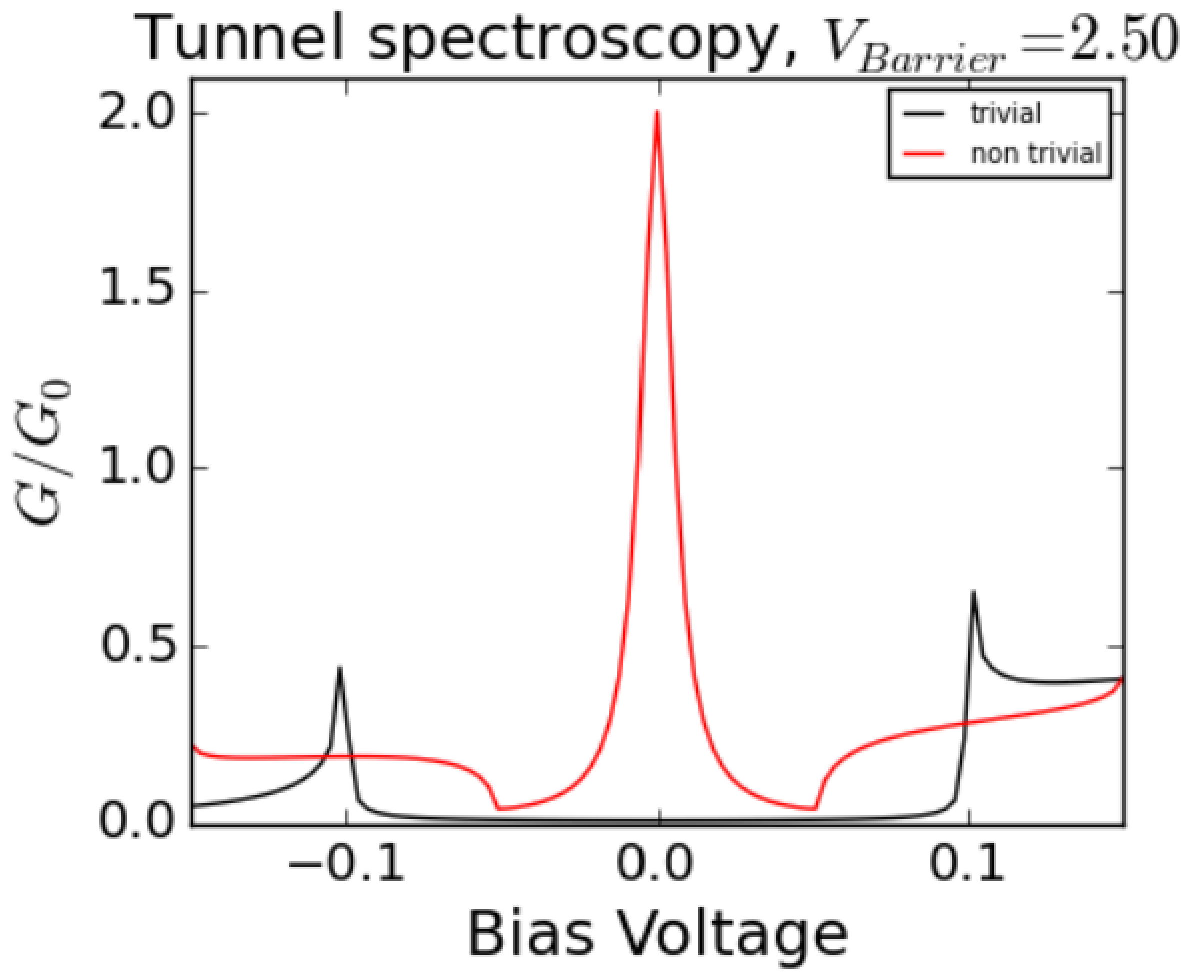


Andreev reflection

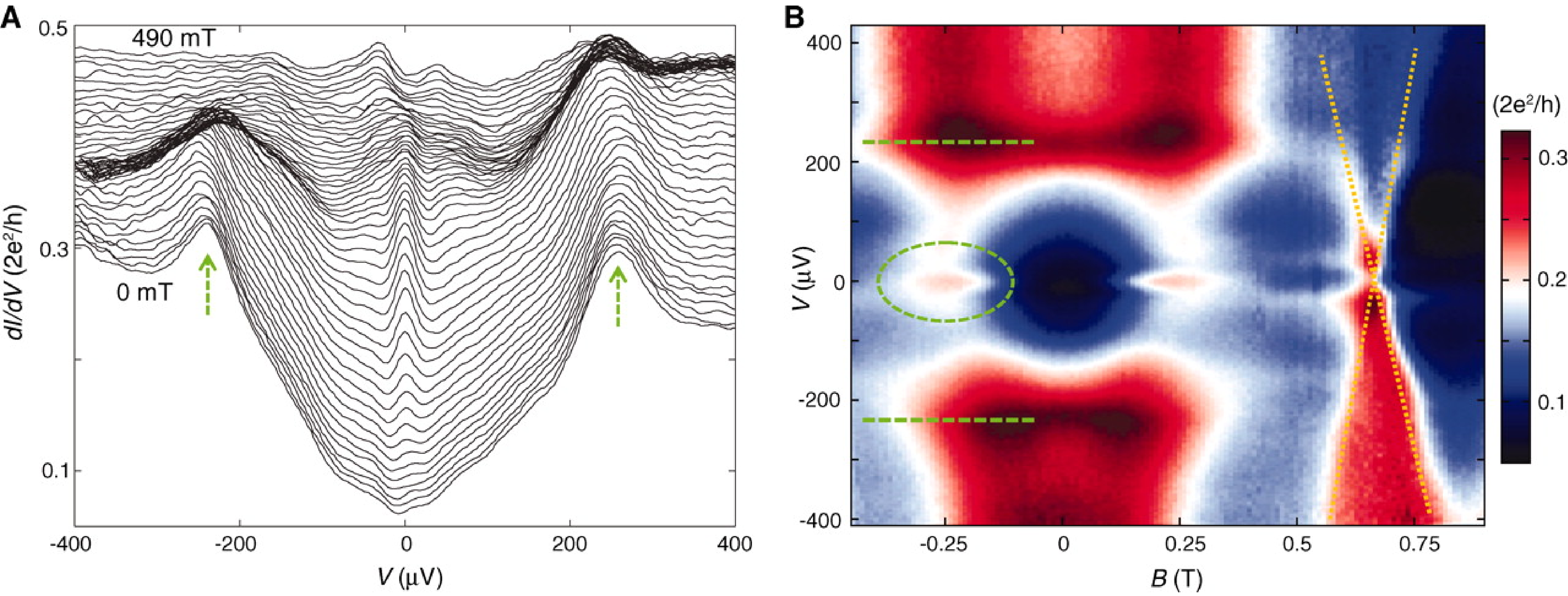


Current 2e ⟶

Conductance



Experimental results



V. Mourik et al., *Signatures of Majorana fermions in hybrid superconductor-semiconductor nanowire devices*, Science, 2012

Extensions

- Wire circuits
- Higher dimensions
- Exchange operations

Thank you for listening